# U-I,Fundamentals of Vibrations

**Oscillating Motions :**

- The study of vibrations is concerned with the oscillating motion of elastic bodies and the force associated with them.
- All bodies possessing mass and elasticity are capable of vibrations.
- Most engineering machines and structures experience vibrations to some degree and their design generally requires consideration of their oscillatory motions.
- Oscillatory systems can be broadly characterized as linear or nonlinear.
- **Linear systems :**

  The principle of superposition holds

  - Mathematical technique available for their analysis are well developed.

# FUNDAMENTALS OF VIBRATIONS

- **Nonlinear systems :**

  The principle of superposition doesn't hold

  - The technique for the analysis of the nonlinear systems are under development (or less well known) and difficult to apply.
- All systems tend to become nonlinear with increasing amplitudes of oscillations.
- There are two general classes of vibrations free and forced.
- ***Free vibration*** takes place when a system oscillates under the action of forces inherent in the system itself due to initial disturbance, and when the externally applied forces are absent.
- The system under free vibration will vibrate at *one or more* of its *natural frequencies*, which are *properties of the dynamical system*, established by its mass and stiffness distribution
- **Forced vibration** takes place under the excitation of external forces is called forced vibration.

# FREE AND FORCED VIBRATIONS

- If excitation is harmonic, the system is forced to vibrate at ***excitation frequency*** . If the frequency of excitation coincide with one of the natural frequencies of the system, a condition of ***resonance*** is encountered and dangerously large oscillations may result, which results in failure of major structures, i.e., bridges, buildings, or airplane wings etc.
- Thus calculation of natural frequencies is of major importance in the study of vibrations.
- Because of friction & other resistances vibrating systems are subjected to ***damping*** to some degree due to dissipation of energy.
- Damping has very ***little effect on natural frequency*** of the system, and hence the calculations for natural frequencies are generally made on the basis of no damping.
- Damping is of great importance in ***limiting the amplitude*** of oscillation at resonance.

# Degrees of Freedom (dof)

- The number of independent co-ordinates required to describe the motion of a system is termed as degrees of freedom.
- *For example*
  *Particle - **3 dof** (positions)*
  *Rigid body -**6 dof***
  *(**3**-positions and **3-**orientations)*
  *Continuous elastic body - **infinite dof***
  *(three positions to each particle of the body).*
- If part of such continuous elastic bodies may be assumed to be rigid (or lumped) and the system may be considered to be dynamically equivalent to one having finite dof (or lumped mass systems).
- Large number of vibration problems can be analyzed with sufficient accuracy by reducing the system to one having a few dof.

# VIBRATION MEASUREMENT TERMINOLOGY

- **Peak value :** Indicates the maximum response of a vibrating part. It also places a limitation on the space requirement.
- **Average value :** Indicates a steady or static value (somewhat like the DC level of an electrical current) and it is defined as

$$\bar{x} = \lim_{T \to \infty} (1/T) \int_0^T x(t)\,dt \qquad (1.1)$$

where $x(t)$ is the displacement, and $T$ is the time span (for example time period)

- For a complete cycle of sine wave,

$$x(t) = A \sin \omega t : \bar{x} = \frac{1}{2\pi} \int_0^{2\pi} A \sin \omega t\,dt = \frac{A}{2\pi} \left[ \frac{\cos \omega t}{\omega} \right]_0^{2\pi} = \frac{A}{2\pi\omega} [1.0 - 1.0] = 0 \qquad (1.2)$$

# Mean and mean square value

- For half cycle of the sine wave :

$$\bar{x} = 1/\pi \int_0^{\pi} A \sin \omega t \, dt = \frac{A}{\pi}\left[\frac{\cos \omega t}{\omega}\right]_0^{\pi} = \frac{A}{\pi\omega}\left[1-(-1)\right] = 2A/\pi = 0.637A$$

where *A* is the amplitude of the displacement.

- **Mean square value :** Square of the displacement generally is associated with the energy of the vibration for which the mean square value is a measure and is defined as

$$< x^2 >= \lim_{T\to\infty}(1/T)\int_0^T x^2(t)\,dt$$

For a complete cycle of sine wave $x(t) = A\sin\omega t$ we have

$$< x^2 >= \lim_{T\to\infty}\left(\frac{A^2}{T}\right)\int_0^T \frac{(1-\cos 2\omega t)}{2}\,dt = \lim_{T\to\infty}\left[\frac{A^2}{2} - \frac{A^2}{2T}\frac{\sin 2\omega t}{2\omega}\Big|_0^T\right] = \frac{A^2}{2} - \frac{A^2}{2}\lim_{T\to\infty}\left(\frac{\sin 2\omega T}{2\omega T}\right) = \frac{A^2}{2}$$

# Root mean square value (rms)

- **Root mean square value (rms) :** This is the square root of the mean square value.
- For example **:** for a complete sine wave

$$x_{rms} = \left[\langle x^2 \rangle\right]^{\frac{1}{2}} = \left[\frac{A}{2}\right]^{\frac{1}{2}} = 0.707A$$

# VIBRATION TERMINOLOGY

**Oscillatory Motion**

- Repeat itself regularly for example pendulum of a wall clock
- Display irregularity for example earthquake
- **Periodic Motion :** This motion repeats at equal interval of time *T*.
- **Period of Oscillatory :** The time taken for one repetition is called period.
- **Frequency -** $f = \frac{1}{T}$ It is defined reciprocal of time period.
- The condition of periodic motion is

$$x(t+T) = x(t) \qquad (1.10)$$

where motion is designated by time function *x*(*t*) .

# HARMONIC MOTION

**Harmonic motion**

- Simplest form of periodic motion is harmonic motion and it is called simple harmonic motion (SHM). It can be expressed as

$$x = A \sin 2\pi \frac{t}{T} \qquad (1.11)$$

where *A* is the amplitude of motion.

Harmonic motion is often represented by projection on line of a point that is moving on a circle at constant speed.

n, *t* is the time instant and *T* is the period of motion.

# SIMPLE HARMONIC MOTION

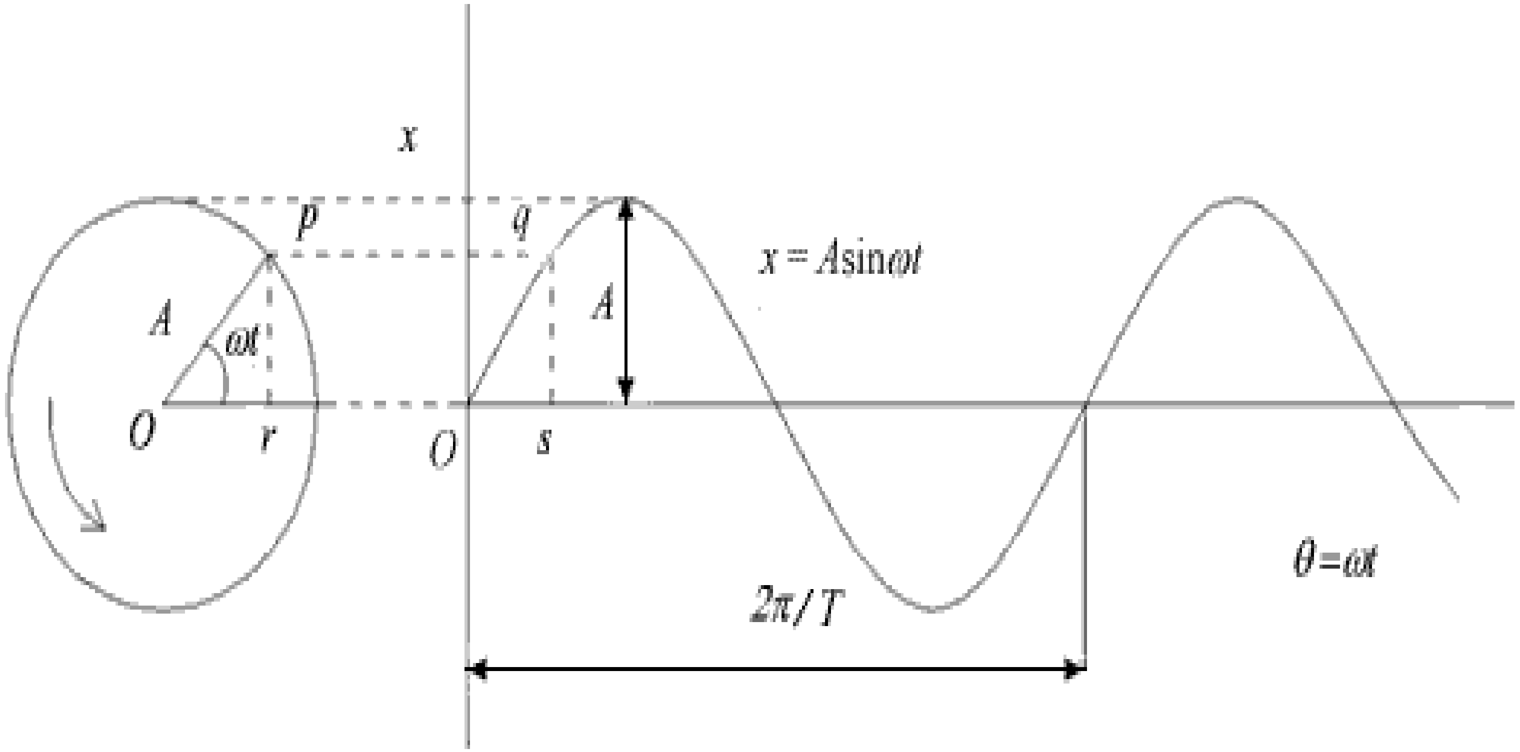



Figure 1.1: The Simple Harmonic Motion

# SIMPLE HARMONIC MOTION

From Figure 1.1 , we have

$$x = pr = qs = A\sin\omega t$$

where $x$ is the displacement and $\omega$ the circular frequency in rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad (1.12)$$

where $T$ is the period (sec) and $f$ is the frequency (cycle/sec) of the harmonic motion.

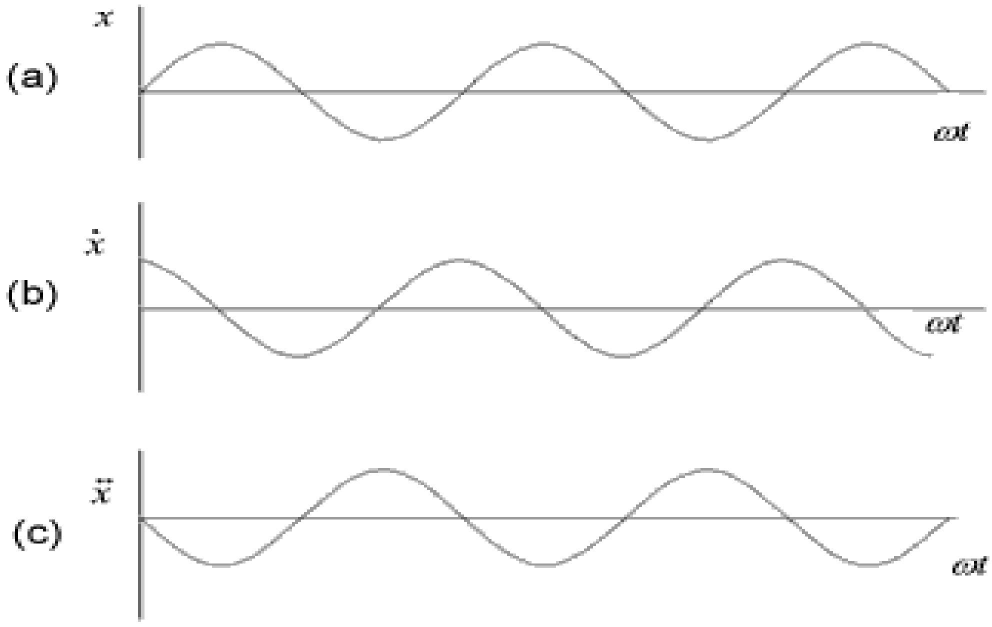
- The SHM repeats itself in $2\pi$ radians.
- Displacement can be expressed as $x = A\sin\omega t$ (1.13)

  velocity can be expressed as $\dot{x} = \omega A\cos\omega t = \omega A\sin(\omega t + \pi/2)$ (1.14)

  acceleration can be written as $\ddot{x} = -\omega^2 A\sin\omega t = \omega^2 A\sin(\omega t + \pi)$ (1.15)

## DISPACEMENT,VELOCITY AND ACCELERATION

- Equations (1.12) to (1.14) are plotted in Figure 1.2




- Figure 1.2 : Variation of displacement, velocity and acceleration with nondimensional time.

# SIMPLE HARMONIC MOTION

- It should be noted from equations (1.12-1.14) that when displacement is a SHM the velocity and acceleration are also harmonic motion with same frequency of oscillation (i.e. displacement). However, lead in phases occurs by $90^0$ and $180^0$ respectively with respect to the displacement as shown in Figure 1.2.
- From equations (1.12) and (1.14) we find

(1.15)

- In harmonic motion acce $\ddot{x} = -\omega^2 x$ oportional to the displacement and is directed towards the origin.
- The Newton's second law of motion states that the acceleration is proportional to the force. Hence for a spring (linear), we write

(1.16)

- where $F_S$ is the $F_s = kx$ orce and $k$ is the stiffness of the spring. It executes harmonic motion as force is proportional to the displacement. (animation)